1. Let $\{X_n\}_{n\geq 0}$ be a Markov chain on state space $S = \{1,2,3\}$ with transition matrix
$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$
and initial distribution μ.

   (a) Draw a graph that represents the above Markov chain.

   (b) Find p_{11}^n

2. Let $\mathbb{T}^2$ be the rooted binary tree. That is, it is an infinite graph with ρ as the distinguished vertex, which comes with a single edge; at every other vertex there are three edges; and there are no closed loops. Place natural weights μ, on the edges. Show that the random walk on $\mathbb{T}^2, \mu$ is transient.

3. Let X_n be a Markov chain on $S = \{1,2,3,4,5\}$

   with the transition probability matrix
$$P = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

   (i) Find the set of transient states, and the irreducible closed set(s) of recurrent states. (ii) Find the probability of eventual absorption in the irreducible closed set(s) of recurrent states